LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **STATISTICS**

FIRST SEMESTER – **NOVEMBER 2018**

MT 1101 – MATHEMATICS FOR STATISTICS

Date: 24-10-2018   Dept. No. [   ]   Max. : 100 Marks
Time: 01:00-04:00

PART A

Answer all the questions: (10 X 2 = 20)

1. Differentiate $\frac{x^3}{3x-2}$ with respect to x.

2. Find $\frac{dy}{dx}$ for $x = a(\theta + sin\ \theta)$; $y = a(1 - cos\ \theta)$.

3. For what value of x is $6x^3 - 2x^2 + 7x - 4$ a decreasing function?

4. Find the points of inflexion on $y = x^3 - 9x^2 + 7x - 6$.

5. If $V = (x^2 + y^2 + z^2)^{1/2}$, find $\frac{\partial V}{\partial x}$.

6. Using Maclaurin's series, expand $sin\ x$ as an infinite series.

7. Find the first order partial differential coefficients of $u = cos(7x + 4y)$.

8. Integrate $e^x x^3$ with respect to x.

9. Evaluate $\int \frac{3x^2 dx}{4 + x^3}$.

10. Find $\int_1^3 (2x^2 + x - 4)\, dx$.

PART B

Answer any FIVE questions: (5 X 8 = 40)

11. If $u = log(x^2 + y^2 + z^2)$, prove that $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = \frac{2}{x^2 + y^2 + z^2}$.

12. Find the points of inflexion on the curve $y = \frac{a^2 x}{x^2 + a^2}$.

13. Find the maximum value of $\frac{log x}{x}$ for positive values of x.

14. Prove that $log(1 + x + x^2) = x + \frac{1}{2}x^2 + \frac{2}{3}x^3 + \frac{1}{4}x^4 + \cdots$

15. Integrate $\frac{e^x}{e^{\frac{x}{2}} - 1}$ with respect to x.

16. Evaluate $\int \frac{x}{x^2 + x + 1}\, dx$.

17. Prove that $\int_0^{\frac{\pi}{2}} \frac{\sqrt{sin x}}{\sqrt{sin x} + \sqrt{cos x}}\, dx = \frac{\pi}{4}$.

18. By transforming into polar coordinates, evaluate $\int_0^{2a} \int_0^{\sqrt{2ax - x^2}} (x^2 + y^2)\, dx dy$.

PART C

Answer any TWO questions: (2 X 20 = 40)

19.(a) If $f(x) = x^3 + x^2 + x - 1$, simplify $f(x + 1) - 3f(x) + 2f(x - 1)$.

   (b) If $y = \cos x \cos 2x$, find $\frac{dy}{dx}$.

   (c) Prove that the tangents to the curve $y = x^2 - 5x + 6$ at the points (2,0) and (3,0) cut at

   right angles. (7+3+10)

20.(a) If $u = \frac{1}{r}$ and $r^2 = (x - a)^2 + (y - b)^2 + (z - c)^2$, prove that $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = 0$.

   (b) Find the maxima and minima of the function $2x^3 - 3x^2 - 36x + 10$. (10+10)

21. (a) Prove that $\int_0^{\frac{\pi}{2}} \log \sin x \, dx = \frac{\pi}{2} \log \left(\frac{1}{2}\right)$.

   (b) Verify Euler's theorem when $u = x^3 - 3x^2 y + 3xy^2 + y^3$. (10+10)

22. (a) Evaluate $\iint (x^2 + y^2) \, dx dy$ over the region for which $x, y \geq 0$ and $x + y \leq 1$.

   (b) Evaluate $\int \frac{dx}{(x+1)(\sqrt{x^2+x+1})}$. (10+10)

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